

Profit function

A company is interested in estimating its profit for various demand levels.

For a demand of 1000 units, the company is making a profit of \$4000 and has a marginal profit of -0.5 \$/unit.

$P(1000)$

$P'(1000)$

If one makes the assumption that the marginal profit remains constant, then the profit function would be linear. The equation of this line is:

$$y = -0.5q + 4500$$

from $y = mq + b$ take $m = -0.5$
use $y = 4000$ when $q = 1000$ solve b

$L_{1000}(q)$
↑
emphasis special
q value

for a demand q and a profit P . We call this the linearization of the profit for a demand of 1000 units.

$$y = P(1000) + P'(1000)(q - 1000)$$

For demand levels that are reasonably close to 1000 units, we can use the the above linearization to approximate the profit.

For example, if the demand increases by 50 units, then the profit will approximately be:

$$P(1050) \approx L_{1000}(1050)$$

actual profit = $4000 - 0.5(1050 - 1000)$
= 3975

And for a decrease of 30 units, the profit will approximately be:

$$P(970) \approx L_{1000}(970)$$

$$= 4000 - 0.5(970 - 1000)$$

0.5(970 + 1000)

General case

Given a differentiable function, we would like to estimate it at several points.

For a given point a on the x -axis, the values of

$f(a)$ and $f'(a)$ are known.

If one makes the assumption that

slope of $f(x)$

remains constant

then the function would be linear. The equation of this line is:

$$y = f(a) + f'(a)(x - a)$$

$$L_a(x) = f(a) + f'(a)(x - a)$$

For x value close to a , we can use the line as an approximation for the

function value

We write:

$$f(x) \approx L_a(x)$$

$$f(x) \approx f(a) + f'(a)(x - a)$$

The function $L_a(x)$ is called the

linearization of f at a

(near a ?)